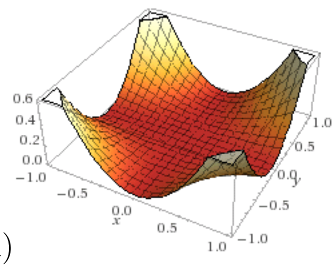
# MATH 105 101 Midterm 1 Sample 2

1. (15 marks) Let $z = f(x, y) = x^2y^2$.

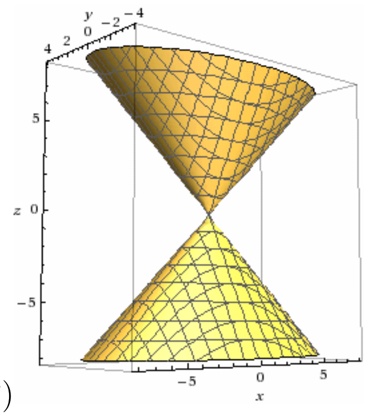
   (a) (5 marks) Compute <u>all</u> second-order partial derivatives of $f$.

   (b) (4 marks) Sketch the level curves $f(x, y) = z_0$ with $z_0 = 0$ and $z_0 = 1$.

   (c) (1 mark) Which of the following renderings represents the graph of the surface?




(A)




(B)

   (d) (3 marks) Find an equation of the plane $\mathcal{P}$ passing through the point $(2, -3, f(2, -3))$ with a normal vector $\mathbf{n} = \langle -1, 3, 2 \rangle$. *Simplify your answer.*

   (e) (2 marks) Does the equation $2x - 6y - 4z = -122$ describe the same plane in (d)? Justify your answer.

2. (5 marks)

   (a) (2 marks) Let $f(x, y) = \ln(9 - x^2 - y^2)$. Sketch the domain of $f$ in the $xy$-plane.

   (b) (3 marks) Show or disprove that there exists a function $g$ which has continuous partial derivatives of all orders such that:

$$g_x = 9998x^{9998}y \text{ and } g_y = x^{9999}.$$

3. (10 marks) Let $R$ be the semicircular region $\{x^2 + y^2 \leq 9, y \geq 0\}$. Find the maximum and minimum values of the function

$$f(x, y) = x^2 + y^2 - 4y$$

   on the *boundary of the region* $R$.

4. (10 marks) Find *all* critical points of the following function:

$$f(x, y) = 3x^2 - 6xy + y^3 - 9y$$

   Classify each point as a local minimum, local maximum, or saddle point.

5. (10 marks) A company wishes to build a new warehouse. It should be situated on the northeast quarter of the Oval, an expressway whose shape is given by the equation:

$$\frac{x^2}{9} + \frac{y^2}{16} = 1.$$

Here $x$ and $y$ are measured in kilometers. From the company's point of view, the desirability of a location on the Oval is measured by the sum of its horizontal and vertical distances from the origin. The larger the sum is, the more desirable the location is. Using the method of Lagrange multipliers, find the location on the Oval that is the most desirable to the company. Clearly state the objective function and the constraint. *You are not required to justify that the solution you obtained is the absolute maximum.* **A solution that does not use the method of Lagrange multipliers will receive no credit, even if it is correct.**